MECHANICAL STRESSES IN SUPERCONDUCTING QUADRUPOLES

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Summary

A solution has been found for stresses in a structural composite that models a shell type superconducting quadrupole. The composite consists of three nested hollow cylinders: the innermost cylinder represents the region of the bore tube, the middle cylinder the region of superconductor, and the outermost cylinder the region of the collars. Under zero stress a distribution of current is chosen to give a pure quadrupole field. Subsequent effects of prestress, cooldown and excitation on the state of stress are determined. Each region is characterized by two elastic constants, one thermal constant and one pretension constant. Two different cases are used to produce a pure quadrupole field: (A) two sheet currents nested between the innermost and middle cylinder each with a surface current density varying as cosine two theta; (B) a thick cosine two theta current distribution in the middle region. Numerical results are given for a beam line quadrupole.

Equation for Elastic Displacements

If \vec{u} is the displacement vector then (1)

$$\nabla \times \nabla \times \vec{u} - 2 \frac{1-v}{1-2v} \nabla (\nabla \cdot \vec{u}) = 2 \frac{1+v}{E} \vec{J} \times \vec{B}, \quad (1)$$

where E is Young's modulus, v is Poisson's ratio, \vec{J} is the current density and \vec{B} is the magnetic induction. If the case of sheet current excitation is used then the RHS of Eq. (1) is zero.



Generalized Plane Strain

For simplicity consider only the case for which $u_z = \epsilon_{zz}z$ with $\epsilon_{zz} = \text{constant}$. The remaining components are to be considered functions of (r, θ) only. This is consistent with an excitation in which J_z is the only component of current density. Hence, Eq. (1) becomes

$$\frac{1}{r} \frac{\partial}{\partial \theta} \left[\frac{1}{r} \frac{\partial}{\partial r} (ru_\theta) - \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right] - \beta \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} (ru_r) + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} \right] = -2 \frac{1+\nu}{E} J_z B_\theta, \quad (2)$$

$$-\frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} (ru_\theta) - \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right] - \beta \frac{1}{r} \frac{\partial}{\partial \theta} \left[\frac{1}{r} \frac{\partial}{\partial r} (ru_r) + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} \right] = 2 \frac{1+\nu}{E} J_z B_r, \quad (3)$$

where

$$\beta = 2 \frac{1-\nu}{1-2\nu}. \quad (4)$$

Magnetic Quantities of InterestCurrent Sheets (Model A):

A pure quadrupole may be generated with two sheet currents each varying as cosine two theta. Thus

$$i_z = \begin{cases} i_b \\ i_c \end{cases} \cos 2\theta. \quad \begin{matrix} (r=b) \\ (r=c) \end{matrix} \quad (\text{abA/cm}) \quad (5)$$

The vector potential corresponding to this excitation is (emu)

$$A_z = \pi \left\{ \begin{matrix} \left[(1+b^4 r_s^{-4}) \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \right] r^2 \\ \left[b^4 r_s^{-4} \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \right] r^2 + b^3 i_b r^{-2} \\ \left[b^4 r_s^{-4} \frac{i_b}{b} + c^4 r_s^{-4} \frac{i_c}{c} \right] r^2 + (b^3 i_b + c^3 i_c) r^{-2} \end{matrix} \right\} \cos 2\theta, \quad (6)$$

where the top entry is for $r < b$, the middle entry for $b < r < c$, and the bottom entry for $c < r < r_s$. See Fig. 1 for geometrical details.

The magnetic induction is given by taking the curl of Eq. (6)

$$B_r = -2\pi \left\{ \begin{array}{l} \left[(1+b^4 r_s^{-4}) \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \right] r \\ b^4 r_s^{-4} \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \end{array} \right\} r + b^3 i_b r^{-3} \sin 2\theta \quad (7)$$

$$\left[b^4 r_s^{-4} \frac{i_b}{b} + c^4 r_s^{-4} \frac{i_c}{c} \right] r + (b^3 i_b + c^3 i_c) r^{-3},$$

$$B_\theta = -2\pi \left\{ \begin{array}{l} \left[(1+b^4 r_s^{-4}) \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \right] r \\ b^4 r_s^{-4} \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \end{array} \right\} r - b^3 i_b r^{-3} \cos 2\theta \quad (8)$$

$$\left[b^4 r_s^{-4} \frac{i_b}{b} + c^4 r_s^{-4} \frac{i_c}{c} \right] r - (b^3 i_b + c^3 i_c) r^{-3}.$$

In order to calculate the forces one needs the average field at the current sheets. Thus

$$B_r = -2\pi \left\{ \begin{array}{l} (1+b^4 r_s^{-4}) i_b + \frac{b}{c} (1+c^4 r_s^{-4}) i_c \\ (1+c^4 r_s^{-4}) (b^3 c^{-3} i_b + i_c) \end{array} \right\} \begin{array}{l} \text{(r=b)} \\ \text{(r=c)} \end{array} \quad (9)$$

$$B_\theta = -2\pi \left\{ \begin{array}{l} b^4 r_s^{-4} i_b + \frac{b}{c} (1+c^4 r_s^{-4}) i_c \\ -\frac{b^3}{c^3} (1-c^4 r_s^{-4}) i_b + c^4 r_s^{-4} i_c \end{array} \right\} \begin{array}{l} \text{(r=b)} \\ \text{(r=c)} \end{array} \quad (10)$$

Hence the force per unit area of the current sheet is given by (dynes/cm²)

$$f_r = \pi \left\{ \begin{array}{l} b^4 r_s^{-4} i_b^2 + \frac{b}{c} (1+c^4 r_s^{-4}) i_b i_c \\ -\frac{b^3}{c^3} (1-c^4 r_s^{-4}) i_b i_c + c^4 r_s^{-4} i_c^2 \end{array} \right\} (1+\cos 4\theta) \quad (11)$$

$$f_\theta = -\pi \left\{ \begin{array}{l} (1+b^4 r_s^{-4}) i_b^2 + \frac{b}{c} (1+c^4 r_s^{-4}) i_b i_c \\ (1+c^4 r_s^{-4}) (\frac{b^3}{c^3} i_b i_c + i_c^2) \end{array} \right\} \sin 4\theta . \quad (12)$$

The magnetic energy stored in a unit length of the quadrupole is given by

$$W_B = \frac{1}{2} \int A_z i_z r d\theta , \quad (13)$$

which when contributions from each shell are added gives

$$W_B = \frac{1}{2} \pi^2 \left\{ b^2 (1+b^4 r_s^{-4}) i_b^2 + 2 \frac{b^3}{c} (1+c^4 r_s^{-4}) i_b i_c + c^2 (1+c^4 r_s^{-4}) i_c^2 \right\} . \quad (\text{ergs/cm}) \quad (14)$$

If $\vec{\tau}$ designates the Maxwell stress tensor then in utilizing the virial theorem⁽²⁾ one needs the projection of the outward radial traction on the radius vector. This becomes

$$\vec{r} \cdot \vec{\tau} \cdot \hat{n} = \frac{1}{8\pi} (B_r^2 - B_\theta^2) r . \quad (15)$$

Evaluating the integral in the virial theorem gives

$$\int_{r=r_s} \vec{r} \cdot \vec{\tau} \cdot \hat{n} r d\theta = 2\pi^2 (b^3 i_b + c^3 i_c)^2 r_s^{-4} . \quad (16)$$

Finally the surface current densities are chosen so that both the total current and the radial moment of the current are the same as for the thick cosine two theta quadrupole. Hence

$$2\pi (bi_b + ci_c) = J_o \pi (c^2 - b^2) , \quad (17)$$

$$2\pi (b^2 i_b + c^2 i_c) = J_o \frac{2}{3} \pi (c^3 - b^3) , \quad (18)$$

which gives

$$i_b = \frac{1}{6b}(c^2 + cb - 2b^2)J_o, \quad (19)$$

$$i_c = \frac{1}{6c}(2c^2 - cb - b^2)J_o. \quad (20)$$

Equations (8), (19), and (20) may be used to eliminate the current density J_o . Thus

$$B_o' = -\frac{\pi}{3}J_o \left\{ \frac{c^2}{b^2} + \frac{c}{b} \frac{b}{c} \frac{b^2}{c^2} + [2(c^4 - b^4) - bc(c^2 - b^2)]r_s^{-4} \right\}. \quad (21)$$

Thick Cosine Two Theta (Model B):

By definition a thick cosine two theta conductor carries an axial current between two radii (b, c) with a current density that varies as

$$J_z = J_o \cos 2\theta. \quad (22)$$

From this it follows that the vector potential⁽³⁾ corresponding to this excitation is (emu)

$$A_z = \pi J_o \left\{ \begin{array}{l} [\ln \frac{c}{b} + \frac{1}{4}(c^4 - b^4)r_s^{-4}]r^2 \\ [\frac{1}{4} + \ln \frac{c}{r} + \frac{1}{4}(c^4 - b^4)r_s^{-4}]r^2 - \frac{1}{4}b^4 r^{-2} \\ \frac{1}{4}(c^4 - b^4)(r_s^{-4}r^2 + r^{-2}) \end{array} \right\} \cos 2\theta, \quad (23)$$

where the three entries are for the three regions explained in Case A.

The magnetic induction becomes

$$B_r = -2\pi J_o \left\{ \begin{array}{l} [\ln \frac{c}{b} + \frac{1}{4}(c^4 - b^4) r_s^{-4}] r \\ [\frac{1}{4} + \ln \frac{c}{r} + \frac{1}{4}(c^4 - b^4) r_s^{-4}] r - \frac{1}{4} b^4 r^{-3} \\ \frac{1}{4}(c^4 - b^4) (rr_s^{-4} + r^{-3}) \end{array} \right\} \sin 2\theta \quad (24)$$

$$B_r = -2\pi J_o \left\{ \begin{array}{l} [\ln \frac{c}{b} + \frac{1}{4}(c^4 - b^4) r_s^{-4}] r \\ [-\frac{1}{4} + \ln \frac{c}{r} + \frac{1}{4}(c^4 - b^4) r_s^{-4}] r + \frac{1}{4} b^4 r^{-3} \\ \frac{1}{4}(c^4 - b^4) (rr_s^{-4} - r^{-3}) \end{array} \right\} \cos 2\theta \quad (25)$$

The Lorentz force in the region of the conductor is (dynes/cm³)

$$f_r = -J_z B_\theta = \pi J_o^2 \left\{ [-\frac{1}{4} + \ln \frac{c}{r} + \frac{1}{4}(c^4 - b^4) r_s^{-4}] r + \frac{1}{4} b^4 r^{-3} \right\} (1 + \cos 4\theta) , \quad (26)$$

$$f_\theta = J_z B_r = -\pi J_o^2 \left\{ [\frac{1}{4} + \ln \frac{c}{r} + \frac{1}{4}(c^4 - b^4) r_s^{-4}] r - \frac{1}{4} b^4 r^{-3} \right\} \sin 4\theta . \quad (27)$$

The magnetic energy stored in a unit length of the quadrupole is given by

$$W_B = \frac{1}{2} \iint A_z J_z r dr d\theta \quad (28)$$

Using Eqs. (22-23) one has

$$W_B = \frac{1}{16} \pi^2 J_o^2 \left\{ -4b^4 \ln \frac{c}{b} + (c^4 - b^4) [1 + \frac{1}{2}(c^4 - b^4) r_s^{-4}] \right\} . \quad (\text{ergs/cm}) \quad (29)$$

The integral of interest in the virial theorem is from Eq. (15) and Eqs. (24-25)

$$\int_{r=r_s} \vec{r} \cdot \vec{\tau} \cdot \vec{n} r dr d\theta = \frac{1}{8} \pi^2 J_o^2 (c^4 - b^4) r_s^{-4} . \quad (30)$$

Finally the current density J_o is chosen by relating it to the desired central gradient. From Eq. (25)

$$B'_o = -2\pi J_o \left[\ln \frac{c}{b} + \frac{1}{4} (c^4 - b^4) r_s^{-4} \right]. \quad (31)$$

Form of Solution

Equations (11-12) and Eqs. (26-27) indicate that a suitable form for the displacement is

$$u_r = P_o(r) + P_4(r) \cos 4\theta \quad u_\theta = Q_4(r) \sin 4\theta. \quad (32)$$

Substituting into Eqs. (2-3) gives

$$-\beta \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (rP_o) \right] = -\mu [(1-\lambda)r + 4r \ln r - b^4 r^{-3}], \quad (33)$$

$$\begin{aligned} \frac{16}{r^2} P_4 - \beta \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (rP_4) \right] + \frac{4}{r^2} \frac{d}{dr} (rQ_4) - 4\beta \frac{d}{dr} \left(\frac{Q_4}{r} \right) &= \\ -\mu [(1-\lambda)r + 4r \ln r - b^4 r^{-3}], \end{aligned} \quad (34)$$

$$\begin{aligned} -4 \frac{d}{dr} \left(\frac{P_4}{r} \right) + \frac{4\beta}{r^2} \frac{d}{dr} (rP_4) - \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (rQ_4) \right] + \frac{16\beta}{r^2} Q_4 &= \\ = \mu [-(1+\lambda)r + 4r \ln r + b^4 r^{-3}], \end{aligned} \quad (35)$$

where for convenience

$$\mu = \frac{1}{2}\pi J_o^2 \frac{1+\nu}{E}, \quad \lambda = (c^4 - b^4) r_s^{-4} + 4 \ln c. \quad (36)$$

Solutions of the Homogeneous Equations

In general these solutions are of the form

$$P_o = Ar^{-1} + Br \quad P_4 = Cr^p \quad Q_4 = Dr^p, \quad (33)$$

where p is found by substituting into Eqs. (34-35) to obtain

$$[16 - \beta(p^2 - 1)]C + 4[p + 1 - \beta(p - 1)]D = 0, \quad (38)$$

$$4[-p + 1 + \beta(p + 1)]C - [p^2 - 1 - 16\beta]D = 0. \quad (39)$$

The determinant of the coefficients is

$$\Delta(p) = \beta(p^2 - 9)(p^2 - 25). \quad (40)$$

Setting this equal to zero gives $p = \pm 3, \pm 5$. Hence there are four solutions which must be added together to give

$$P_4 = -D_1 r^3 + \frac{1-2\beta}{2-\beta} D_2 r^{-3} - \frac{3-2\beta}{2-3\beta} D_3 r^5 + D_4 r^{-5}, \quad (41)$$

$$Q_4 = D_1 r^3 + D_2 r^{-3} + D_3 r^5 + D_4 r^{-5}. \quad (42)$$

In addition to solutions of the form given in Eq. (32) another solution is added of the form

$$u_r = \frac{1}{\beta} Gr \ln r \quad u_\theta = Gr\theta, \quad (43)$$

which is utilized in describing pretension.

Displacement

Collecting all the forms together one has after relabeling the constants

$$u_r = \left\{ \begin{array}{l} A_1 r^{-1} \quad +B_1 r \\ \quad +[-C_1 r^3 + \frac{1-2\beta_1}{2-\beta_1} D_1 r^{-3} - \frac{3-2\beta_1}{2-3\beta_1} E_1 r^5 + F_1 r^{-5}] \cos 4\theta. \\ A_2 r^{-1} \quad +B_2 r \\ \quad +[-C_2 r^3 + \frac{1-2\beta_2}{2-\beta_2} D_2 r^{-3} - \frac{3-2\beta_2}{2-3\beta_2} E_2 r^5 + F_2 r^{-5}] \cos 4\theta \\ A_3 r^{-1} + \frac{1}{\beta_3} G_3 r \ln r + B_3 r \\ \quad +[-C_3 r^3 + \frac{1-2\beta_3}{2-\beta_3} D_3 r^{-3} - \frac{3-2\beta_3}{2-3\beta_3} E_3 r^5 + F_3 r^{-5}] \cos 4\theta \end{array} \right\} \quad (44)$$

$$u_\theta = \left\{ \begin{array}{l} [C_1 r^3 + D_1 r^{-3} + E_1 r^5 + F_1 r^{-5}] \sin 4\theta \\ [C_2 r^3 + D_2 r^{-3} + E_2 r^5 + F_2 r^{-5}] \sin 4\theta \\ G_3 r \theta + [C_3 r^3 + D_3 r^{-3} + E_3 r^5 + F_3 r^{-5}] \sin 4\theta \end{array} \right\}. \quad (45)$$

Strain

Using Eqs. (61-63) from Ref. (2) the strains are given by

$$\epsilon_{rr} = \left\{ \begin{array}{l} -A_1 r^{-2} + B_1 \\ \quad + [-3C_1 r^2 - 3\frac{1-2\beta_1}{2-\beta_1} D_1 r^{-4} - 5\frac{3-2\beta_1}{2-3\beta_1} E_1 r^4 - 5F_1 r^{-6}] \cos 4\theta \\ -A_2 r^{-2} + B_2 \\ \quad + [-3C_2 r^2 - 3\frac{1-2\beta_2}{2-\beta_2} D_2 r^{-4} - 5\frac{3-2\beta_2}{2-3\beta_2} E_2 r^4 - 5F_2 r^{-6}] \cos 4\theta \\ -A_3 r^{-2} + \frac{1}{\beta_3} G_3 (1 + \ln r) + B_3 \\ \quad + [-3C_3 r^2 - 3\frac{1-2\beta_3}{2-\beta_3} D_3 r^{-4} - 5\frac{3-2\beta_3}{2-3\beta_3} E_3 r^4 - 5F_3 r^{-6}] \cos 4\theta \end{array} \right\}, \quad (46)$$

$$\epsilon_{\theta\theta} = \left\{ \begin{array}{l} A_1 r^{-2} + B_1 \\ \quad + [3C_1 r^2 + 3\frac{3-2\beta_1}{2-\beta_1} D_1 r^{-4} + 5\frac{1-2\beta_1}{2-3\beta_1} E_1 r^4 + 5F_1 r^{-6}] \cos 4\theta \\ A_2 r^{-2} + B_2 \\ \quad + [3C_2 r^2 + 3\frac{3-2\beta_2}{2-\beta_2} D_2 r^{-4} + 5\frac{1-2\beta_2}{2-3\beta_2} E_2 r^4 + 5F_2 r^{-6}] \cos 4\theta \\ A_3 r^{-2} + G_3 (1 + \frac{1}{\beta_3} \ln r) + B_3 \\ \quad + [3C_3 r^2 + 3\frac{3-2\beta_3}{2-\beta_3} D_3 r^{-4} + 5\frac{1-2\beta_3}{2-3\beta_3} E_3 r^4 + 5F_3 r^{-6}] \cos 4\theta \end{array} \right\}, \quad (47)$$

$$\epsilon_{r\theta} = \left\{ \begin{array}{l} 3C_1 r^2 - 6\frac{1-\beta_1}{2-\beta_1} D_1 r^{-4} + 10\frac{1-\beta_1}{2-3\beta_1} E_1 r^4 - 5F_1 r^{-6} \\ 3C_2 r^2 - 6\frac{1-\beta_2}{2-\beta_2} D_2 r^{-4} + 10\frac{1-\beta_2}{2-3\beta_2} E_2 r^4 - 5F_2 r^{-6} \\ 3C_3 r^2 - 6\frac{1-\beta_3}{2-\beta_3} D_3 r^{-4} + 10\frac{1-\beta_3}{2-3\beta_3} E_3 r^4 - 5F_3 r^{-6} \end{array} \right\} \sin 4\theta. \quad (48)$$

Stress

To obtain the stress invert Eqs. (48-50) of Ref. (2). Thus, after using Eq. (4)

$$\sigma_{rr} = \frac{1}{2} \frac{E}{1+\nu} [\beta \varepsilon_{rr} - (2-\beta) \varepsilon_{\theta\theta} - (2-\beta) \varepsilon_{zz} + (4-3\beta) k], \quad (49)$$

$$\sigma_{\theta\theta} = \frac{1}{2} \frac{E}{1+\nu} [-(2-\beta) \varepsilon_{rr} + \beta \varepsilon_{\theta\theta} - (2-\beta) \varepsilon_{zz} + (4-3\beta) k], \quad (50)$$

$$\sigma_{r\theta} = \frac{E}{1+\nu} \varepsilon_{r\theta}, \quad (51)$$

where k is the thermal expansion coefficient integrated from room temperature to say 4.2°K . Using the homogeneous contribution to the strain from Eqs. (46-48) the homogeneous contribution to the stress becomes

$$\begin{aligned} \frac{1+\nu}{E} \sigma_{rr} = & \frac{-A_1 r^{-2}}{\frac{1}{2}(2-\beta_1) \varepsilon_{zz} + \frac{1}{2}(4-3\beta_1) k_1 - (1-\beta_1) B_1} \\ & - [3C_1 r^2 + 9 \frac{1-\beta_1}{2-\beta_1} D_1 r^{-4} + 5 \frac{1-\beta_1}{2-3\beta_1} E_1 r^4 + 5F_1 r^{-6}] \cos 4\theta \\ & \frac{-A_2 r^{-2}}{\frac{1}{2}(2-\beta_2) \varepsilon_{zz} + \frac{1}{2}(4-3\beta_2) k_2 - (1-\beta_2) B_2} \\ & - [3C_2 r^2 + 9 \frac{1-\beta_2}{2-\beta_2} D_2 r^{-4} + 5 \frac{1-\beta_2}{2-3\beta_2} E_2 r^4 + 5F_2 r^{-6}] \cos 4\theta \\ & \frac{-A_3 r^{-2} - G_3 (1-\beta_3) (\frac{1}{2} + \frac{1}{\beta_3} \ln r) - \frac{1}{2} (2-\beta_3) \varepsilon_{zz} + \frac{1}{2}(4-3\beta_3) k_3 - (1-\beta_3) B_3}{\frac{1}{2}(2-\beta_3) \varepsilon_{zz} + \frac{1}{2}(4-3\beta_3) k_3 - (1-\beta_3) B_3} \\ & - [3C_3 r^2 + 9 \frac{1-\beta_3}{2-\beta_3} D_3 r^{-4} + 5 \frac{1-\beta_3}{2-3\beta_3} E_3 r^4 + 5F_3 r^{-6}] \cos 4\theta \end{aligned}, \quad (52)$$

$$\begin{aligned}
 & A_1 r^{-2} \quad -\frac{1}{2}(2-\beta_1) \epsilon_{zz} + \frac{1}{2}(4-3\beta_1) k_1 - (1-\beta_1) B_1 \\
 & + [3C_1 r^2 + 3\frac{1-\beta_1}{2-\beta_1} D_1 r^{-4} + 15\frac{1-\beta_1}{2-3\beta_1} E_1 r^4 + 5F_1 r^{-6}] \cos 4\theta \\
 \frac{1+\nu}{E} \sigma_{\theta\theta} = & A_2 r^{-2} \quad -\frac{1}{2}(2-\beta_2) \epsilon_{zz} + \frac{1}{2}(4-3\beta_2) k_2 - (1-\beta_2) B_2 \\
 & + [3C_2 r^2 + 3\frac{1-\beta_2}{2-\beta_2} D_2 r^{-4} + 15\frac{1-\beta_2}{2-3\beta_2} E_2 r^4 + 5F_2 r^{-6}] \cos 4\theta \\
 & A_3 r^{-2} - G_3 \frac{1-\beta_3}{\beta_3} (1 + \frac{1}{2}\beta_3 + \ln r) - \frac{1}{2}(2-\beta_3) \epsilon_{zz} + \frac{1}{2}(4-3\beta_3) k_3 - (1-\beta_3) B_3 \\
 & + [3C_3 r^2 + 3\frac{1-\beta_3}{2-\beta_3} D_3 r^{-4} + 15\frac{1-\beta_3}{2-3\beta_3} E_3 r^4 + 5F_3 r^{-6}] \cos 4\theta, \quad (53)
 \end{aligned}$$

$$\frac{1+\nu}{E} \sigma_{r\theta} = \left\{ \begin{array}{l} 3C_1 r^2 - 6\frac{1-\beta_1}{2-\beta_1} D_1 r^{-4} + 10\frac{1-\beta_1}{2-3\beta_1} E_1 r^4 - 5F_1 r^{-6} \\ 3C_2 r^2 - 6\frac{1-\beta_2}{2-\beta_2} D_2 r^{-4} + 10\frac{1-\beta_2}{2-3\beta_2} E_2 r^4 - 5F_2 r^{-6} \\ 3C_3 r^2 - 6\frac{1-\beta_3}{2-\beta_3} D_3 r^{-4} + 10\frac{1-\beta_3}{2-3\beta_3} E_3 r^4 - 5F_3 r^{-6} \end{array} \right\} \sin 4\theta \quad (54)$$

Particular Solution

The particular solution of Eq. (33) may be found by integrating and dropping those terms already included in the homogeneous form. Thus

$$P_O = \frac{\mu}{\beta} \left[-\frac{1}{8}(2+\lambda) r^3 + \frac{1}{2} r^3 \ln r + \frac{1}{2} b^4 r^{-1} \ln r \right] \quad (55)$$

The contribution to the particular solutions (P_4, Q_4) of Eqs. (34-35) arising from a term on the RHS of the form r^{-3} may be found by letting $\begin{pmatrix} P_4 \\ Q_4 \end{pmatrix} = \begin{pmatrix} C \\ D \end{pmatrix} r^{-1}$. Thus

$$16C+8\beta D = \mu b^4 \quad (56)$$

$$8C+16\beta D = \mu b^4 \quad (57)$$

or

$$C = \frac{1}{24}\mu b^4 \quad D = \frac{1}{24}\frac{\mu}{\beta} b^4. \quad (58)$$

The remaining contribution to the particular solution (P_4, Q_4) arising from the terms on the RHS proportional to r and to $rlnr$ may be found by assuming solutions of the form

$$P_4 = (R+Slnr+Tln^2r)r^3, \quad (59)$$

$$Q_4 = (U+Vlnr+Wln^2r)r^3. \quad (60)$$

Substituting into Eq. (34) gives

$$\begin{aligned} & [(16-8\beta)(R+U)-6\beta S+4(1-\beta)V-2\beta T]r \\ & + [(16-8\beta)(S+V)-12\beta T+8(1-\beta)W]rlnr \\ & + (16-8\beta)(T+W)rln^2r = -\mu[(1-\lambda)r+4rlnr]. \end{aligned} \quad (61)$$

Substituting Eq. (59-60) into Eq. (35) gives

$$\begin{aligned} & [(-8+16\beta)(R+U)-4(1-\beta)S-6V-2W]r \\ & + [(-8+16\beta)(S+V)-8(1-\beta)T-12W]rlnr \\ & + (-8+16\beta)(T+W)rln^2r = \mu[-(1+\lambda)r+4rlnr] \end{aligned} \quad (62)$$

In this substitution the term on the RHS of Eqs. (34-35) proportional to r^{-3} has been deleted since this contribution has already been found.

Setting

$$T+W = 0 \quad (63)$$

eliminates the term in rln^2r from Eqs. (61-62). Equating the coefficients of $rlnr$ on each side of Eqs. (61-62) gives

$$(16-8\beta)(S+V)-12\beta T+8(1-\beta)W = -4\mu, \quad (64)$$

$$(-8+16\beta)(S+V)-8(1-\beta)T-12W = 4\mu. \quad (65)$$

Solving Eqs. (63-65) simultaneously gives

$$T = \frac{1}{6}(1+\beta)\frac{\mu}{\beta}, \quad W = -\frac{1}{6}(1+\beta)\frac{\mu}{\beta}, \quad (66)$$

$$S+V = \frac{1}{12}(1-\beta)\frac{\mu}{\beta}. \quad (67)$$

Equating the coefficients of r on each side of Eqs. (61-62) gives

$$[(16-8\beta)(R+U)-6\beta S+4(1-\beta)V-2\beta T] = -\mu(1-\lambda), \quad (68)$$

$$[(-8+16\beta)(R+U)-4(1-\beta)S-6V-2W] = -\mu(1+\lambda). \quad (69)$$

Solving Eqs. (68-69) simultaneously gives

$$S = -\frac{1}{12}\frac{\mu}{\beta}[\frac{1}{6}(19-11\beta)+(1+\beta)\lambda], \quad V = \frac{1}{12}\frac{\mu}{\beta}[\frac{1}{6}(25-17\beta)+(1+\beta)\lambda], \quad (70)$$

$$R+U = -\frac{1}{48}\frac{\mu}{\beta}[\frac{1}{6}(25+11\beta)+(1-\beta)\lambda]. \quad (71)$$

Since R and U are coefficients of r^3 , a term that is already included in the homogeneous solution, it is possible to choose one relation between R and U arbitrarily. Hence let $U = 0$. Then

$$R = -\frac{1}{48}\frac{\mu}{\beta}[\frac{1}{6}(25+11\beta)+(1-\beta)\lambda], \quad U = 0. \quad (72)$$

Collecting all the contributions gives

$$\begin{aligned} P_4 &= \frac{\mu}{\beta} \left\{ \frac{1}{24} \beta b^4 r^{-1} - \frac{1}{48} [\frac{1}{6}(25+11\beta)+(1-\beta)\lambda] r^3 \right. \\ &\quad \left. - \frac{1}{12} [\frac{1}{6}(19-11\beta)+(1+\beta)\lambda] r^3 \ln r + \frac{1}{6}(1+\beta) r^3 \ln^2 r \right\}, \end{aligned} \quad (73)$$

$$\begin{aligned} Q_4 &= \frac{\mu}{\beta} \left\{ \frac{1}{24} b^4 r^{-1} + \frac{1}{12} [\frac{1}{6}(25-17\beta)+(1+\beta)\lambda] r^3 \ln r \right. \\ &\quad \left. - \frac{1}{6}(1+\beta) r^3 \ln^2 r \right\}. \end{aligned} \quad (74)$$

Displacement

The particular solution exists only in the region of conductor ($b < r < c$). If this part of the solution is thought of as an additional displacement to be added to the homogeneous forms previously found, then

$$\Delta u_r = P_0 + P_4 \cos 4\theta \quad \Delta u_\theta = Q_4 \sin 4\theta, \quad (75)$$

where P_0 , P_4 , Q_4 are given in Eq. (55) and Eqs. (73-74).

Strain

Using Eqs. (61-63) from Ref. (2) the incremental strains are given by

$$\begin{aligned} \Delta \varepsilon_{rr} = \frac{\mu}{\beta} \left\{ -\frac{1}{8}(2+3\lambda)r^2 + \frac{3}{2}r^2 \ln r + \frac{1}{2}b^4 r^{-2} - \frac{1}{2}b^4 r^{-2} \ln r \right. \\ \left. + \left[-\frac{1}{24}\beta b^4 r^{-2} - \frac{1}{48}\left[\frac{1}{6}(15\lambda-11\beta)+(7+\beta)\lambda\right]r^2 \right. \right. \\ \left. \left. - \frac{1}{24}[11-19\beta+6(1+\beta)\lambda]r^2 \ln r + \frac{1}{2}(1+\beta)r^2 \ln^2 r \right] \cos 4\theta \right\}, \end{aligned} \quad (76)$$

$$\begin{aligned} \Delta \varepsilon_{\theta\theta} = \frac{\mu}{\beta} \left\{ -\frac{1}{8}(2+\lambda)r^2 + \frac{1}{2}r^2 \ln r + \frac{1}{2}b^4 r^{-2} \ln r \right. \\ \left. + \left[\frac{1}{24}(4+\beta)b^4 r^{-2} - \frac{1}{48}\left[\frac{1}{6}(25+11\beta)+(1-\beta)\lambda\right]r^2 \right. \right. \\ \left. \left. + \frac{1}{24}[27-19\beta+6(1+\beta)\lambda]r^2 \ln r - \frac{1}{2}(1+\beta)r^2 \ln^2 r \right] \cos 4\theta \right\}, \end{aligned} \quad (77)$$

$$\begin{aligned} \Delta \varepsilon_{r\theta} = \frac{\mu}{\beta} \left\{ -\frac{1}{24}(1+2\beta)b^4 r^{-2} + \frac{1}{12}\left[\frac{1}{6}(25-3\beta)+\lambda\right]r^2 \right. \\ \left. + \frac{1}{12}\left[\frac{51}{6}(1-\beta)+3(1+\beta)\lambda\right]r^2 \ln r - \frac{1}{2}(1+\beta)r^2 \ln^2 r \right\} \sin 4\theta. \end{aligned} \quad (78)$$

Stress

The incremental stress is related to the incremental strain using Eqs. (49-51) after dropping the terms in ϵ_{zz} and k . Thus

$$\Delta\sigma_{rr} = \frac{E}{1+\nu} \left[\frac{1}{2}\beta\Delta\epsilon_{rr} - \frac{1}{2}(2-\beta)\Delta\epsilon_{\theta\theta} \right] \quad (79)$$

$$\Delta\sigma_{\theta\theta} = \frac{E}{1+\nu} \left[-\frac{1}{2}(2-\beta)\Delta\epsilon_{rr} + \frac{1}{2}\beta\Delta\epsilon_{\theta\theta} \right], \quad (80)$$

$$\Delta\sigma_{r\theta} = \frac{E}{1+\nu}\Delta\epsilon_{r\theta}. \quad (81)$$

Applying these relations to Eqs. (76-78) gives

$$\begin{aligned} \Delta\sigma_{rr} = \frac{E}{1+\nu} \cdot \frac{\mu}{\beta} & \left\{ \frac{1}{8}[2(1-\beta)+(1-2\beta)\lambda]r^2 - \frac{1}{2}(1-2\beta)r^2\ln r \right. \\ & + \frac{1}{4}\beta b^4 r^{-2} - \frac{1}{2}b^4 r^{-2}\ln r \\ & + \left[-\frac{1}{24}(4-\beta)b^4 r^{-2} + \frac{1}{96}[\frac{1}{3}(25-77\beta)+2(1-5\beta)\lambda]r^2 \right. \\ & \left. \left. - \frac{1}{8}[9(1-\beta)+2(1+\beta)\lambda]r^2\ln r + \frac{1}{2}(1+\beta)r^2\ln^2 r \right] \cos 4\theta \right\} , \end{aligned} \quad (82)$$

$$\begin{aligned} \Delta\sigma_{\theta\theta} = \frac{E}{1+\nu} \cdot \frac{\mu}{\beta} & \left\{ \frac{1}{8}[2(1-\beta)+(3-2\beta)\lambda]r^2 - \frac{1}{2}(3-2\beta)r^2\ln r \right. \\ & - \frac{1}{4}(2-\beta)b^4 r^{-2} + \frac{1}{2}b^4 r^{-2}\ln r \\ & + \left[\frac{1}{8}\beta b^4 r^{-2} + \frac{1}{96}[\frac{1}{3}(151-99\beta)+2(7-3\beta)\lambda]r^2 \right. \\ & \left. + \frac{1}{48}[22(1-\beta)+12(1+\beta)\lambda]r^2\ln r - \frac{1}{2}(1+\beta)r^2\ln^2 r \right] \cos 4\theta \right\}. \end{aligned} \quad (83)$$

$$\begin{aligned} \Delta\sigma_{r\theta} = \frac{E}{1+\nu} \cdot \frac{\mu}{\beta} & \left\{ -\frac{1}{24}(1+2\beta)b^4 r^{-2} + \frac{1}{12}[\frac{1}{6}(25-3\beta)+\lambda]r^2 \right. \\ & + \left. \frac{1}{12}[\frac{51}{6}(1-\beta)+3(1+\beta)\lambda]r^2\ln r - \frac{1}{2}(1+\beta)r^2\ln^2 r \right\} \sin 4\theta. \end{aligned} \quad (84)$$

Boundary Conditions

Within the innermost cylinder ($a < r < b$) and within the outermost cylinder ($c < r < d$) Eqs. (44-45) and Eqs. (52-54) are complete expressions for the displacement and stress. Inside the middle cylinder and for Model B Eqs. (44-45) must be supplemented by Eqs. (55, 73-75) to give the general expression for displacement. In addition, Eqs. (52-54) must be augmented by Eqs. (82-84) to give the complete form for stress. Using the complete forms the boundary conditions are as follows.

At $r=a$, the innermost radius

$$\sigma_{rr}^{(+)} = \sigma_{r\theta}^{(+)} = 0, \quad (85)$$

At $r=b$,

$$\sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} = -f_r(b), \quad (86)$$

$$\sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} = -f_\theta(b), \quad (87)$$

$$u_r^{(+)} - u_r^{(-)} = 0, \quad (88)$$

$$u_\theta^{(+)} - u_\theta^{(-)} = 0, \quad (89)$$

At $r=c$

$$\sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} = -f_r(c), \quad (90)$$

$$\sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} = -f_\theta(c), \quad (91)$$

$$u_r^{(+)} - u_r^{(-)} = 0, \quad (92)$$

$$u_\theta^{(+)}(2\pi) - u_\theta^{(+)}(0) = c\alpha, \quad (93)$$

and, after removing term proportional to θ

$$u_\theta^{(+)} - u_\theta^{(-)} = 0, \quad (94)$$

At $r=d$, the outermost radius

$$\sigma_{rr}^{(-)} = \sigma_{r\theta}^{(-)} = 0. \quad (95)$$

Note that f_r and f_θ are given by Eqs. (11-12) and may be written as

$$f_r = f_{ro} + f_{r4} \cos 4\theta, \quad (96)$$

$$f_\theta = f_{\theta 4} \sin 4\theta. \quad (97)$$

Since the normal stress σ_{rr} and the radial displacement u_r involve both isotropic terms and terms proportional to $\cos 4\theta$ there are 19 relations to be obtained from Eqs. (85-95). An examination of Eq. (52) indicates that there are twenty unknowns ($A_1 B_1 C_1 D_1 E_1 F_1 A_2 B_2 C_2 D_2 E_2 F_2 A_3 G_3 B_3 C_3 D_3 E_3 F_3 \epsilon_{zz}$). The virial theorem will be used to supply the last relation.

In detail Eq. (85) gives

$$\begin{aligned} \frac{E_1}{1+\nu_1} & [-A_1 a^{-2} - (1-\beta_1) B_1 - \frac{1}{2} (2-\beta_1) \epsilon_{zz}] \\ & = -\frac{E_1}{1+\nu_1} \frac{1}{2} (4-3\beta_1) k_1, \end{aligned} \quad (98)$$

$$\frac{E_1}{1+\nu_1} [-3C_1 a^2 - 9 \frac{1-\beta_1}{2-\beta_1} D_1 a^{-4} - 5 \frac{1-\beta_1}{2-3\beta_1} E_1 a^4 - 5F_1 a^{-6}] = 0, \quad (99)$$

$$\frac{E_1}{1+\nu_1} [3C_1 a^2 - 6 \frac{1-\beta_1}{2-\beta_1} D_1 a^{-4} + 10 \frac{1-\beta_1}{2-3\beta_1} E_1 a^4 - 5F_1 a^{-6}] = 0. \quad (100)$$

At $r=b$, Eqs. (86-89) give

$$\begin{aligned} \frac{E_1}{1+\nu_1} & [A_1 b^{-2} + (1-\beta_1) B_1 + \frac{1}{2} (2-\beta_1) \epsilon_{zz}] \\ & + \frac{E_2}{1+\nu_2} [-A_2 b^{-2} - (1-\beta_2) B_2 - \frac{1}{2} (2-\beta_2) \epsilon_{zz}] \\ & = -\frac{E_2}{1+\nu_2} \frac{1}{2} (4-3\beta_2) k_2 + \frac{E_1}{1+\nu_1} \cdot \frac{1}{2} (4-3\beta_1) k_1 - \Delta \sigma_{rro}(b) - f_{ro}(b), \end{aligned} \quad (101)$$

$$\begin{aligned}
 & \frac{E_1}{1+\nu_1} [3C_1 b^2 + 9 \frac{1-\beta_1}{2-\beta_1} D_1 b^{-4} + 5 \frac{1-\beta_1}{2-3\beta_1} E_1 b^4 + 5F_1 b^{-6}] \\
 & + \frac{E_2}{1+\nu_2} [-3C_2 b^2 - 9 \frac{1-\beta_2}{2-\beta_2} D_2 b^{-4} - 5 \frac{1-\beta_2}{2-3\beta_2} E_2 b^4 - 5F_2 b^{-6}] \\
 & = -\Delta\sigma_{rr4}(b) - f_{r4}(b), \tag{102}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{E_1}{1+\nu_1} [-3C_1 b^2 + 6 \frac{1-\beta_1}{2-\beta_1} D_1 b^{-4} - 10 \frac{1-\beta_1}{2-3\beta_1} E_1 b^4 + 5F_1 b^{-6}] \\
 & + \frac{E_2}{1+\nu_2} [3C_2 b^2 - 6 \frac{1-\beta_2}{2-\beta_2} D_2 b^{-4} + 10 \frac{1-\beta_2}{2-3\beta_2} E_2 b^4 - 5F_2 b^{-6}] \\
 & = -\Delta\sigma_{r\theta4}(b) - f_{\theta4}(b), \tag{103}
 \end{aligned}$$

$$-A_1 b^{-1} - B_1 b + A_2 b^{-1} + B_2 b = -\Delta u_{ro}(b), \tag{104}$$

$$\begin{aligned}
 & C_1 b^3 - \frac{1-2\beta_1}{2-\beta_1} D_1 b^{-3} + \frac{3-2\beta_1}{2-3\beta_1} E_1 b^5 - F_1 b^{-5} \\
 & - C_2 b^3 + \frac{1-2\beta_2}{2-\beta_2} D_2 b^{-3} - \frac{3-2\beta_2}{2-3\beta_2} E_2 b^5 + F_2 b^{-5} = -\Delta u_{r4}(b), \tag{105}
 \end{aligned}$$

$$\begin{aligned}
 & -C_1 b^3 - D_1 b^{-3} - E_1 b^5 - F_1 b^{-5} \\
 & + C_2 b^3 + D_2 b^{-3} + E_2 b^5 + F_2 b^{-5} = -\Delta u_{\theta4}(b). \tag{106}
 \end{aligned}$$

At $r=c$ Eqs. (90-93) give

$$\begin{aligned}
 & \frac{E_2}{1+\nu_2} [A_2 c^{-2} + (1-\beta_2) B_2 + \frac{1}{2} (2-\beta_2) \epsilon_{zz}] \\
 & + \frac{E_3}{1+\nu_3} [-A_3 c^{-2} - G_3 (1-\beta_3) (\frac{1}{2} + \frac{1}{\beta_3} \ln c) - (1-\beta_3) B_3 - \frac{1}{2} (2-\beta_3) \epsilon_{zz}] \\
 & = -\frac{E_3}{1+\nu_3} \cdot \frac{1}{2} (4-3\beta_3) k_3 + \frac{E_2}{1+\nu_2} \cdot \frac{1}{2} (4-3\beta_2) k_2 + \Delta\sigma_{rro}(c) - f_{ro}(c), \tag{107}
 \end{aligned}$$

$$\begin{aligned} & \frac{E_2}{1+\nu_2} [3C_2c^2 + 9\frac{1-\beta_2}{2-\beta_2}D_2c^{-4} + 5\frac{1-\beta_2}{2-3\beta_2}E_2c^4 + 5F_2c^{-6}] \\ & + \frac{E_3}{1+\nu_3} [-3C_3c^2 - 9\frac{1-\beta_3}{2-\beta_3}D_3c^{-4} - 5\frac{1-\beta_3}{2-3\beta_3}E_3c^4 - 5F_3c^{-6}] = \Delta\sigma_{rr4}(c) - f_{r4}(c). \end{aligned} \quad (108)$$

$$\begin{aligned} & \frac{E_2}{1+\nu_2} [-3C_2c^2 + 6\frac{1-\beta_2}{2-\beta_2}D_2c^{-4} - 10\frac{1-\beta_2}{2-3\beta_2}E_2c^4 + 5F_2c^{-6}] \\ & + \frac{E_3}{1+\nu_3} [3C_3c^2 - 6\frac{1-\beta_3}{2-\beta_3}D_3c^{-4} + 10\frac{1-\beta_3}{2-3\beta_3}E_3c^4 - 5F_3c^{-6}] = \Delta\sigma_{r\theta4}(c) - f_{\theta4}(c), \end{aligned} \quad (109)$$

$$\begin{aligned} & -A_2c^{-1} \quad -B_2c \\ & + A_3c^{-1} + \frac{1}{\beta_3}cG_3\ln c + B_3c = \Delta u_{ro}(c). \end{aligned} \quad (110)$$

$$\begin{aligned} & C_2c^3 - \frac{1-2\beta_2}{2-\beta_2}D_2c^{-3} + \frac{3-2\beta_2}{2-3\beta_2}E_2c^5 - F_2c^{-5} \\ & - C_3c^3 + \frac{1-2\beta_3}{2-\beta_3}D_3c^{-3} - \frac{3-2\beta_3}{2-3\beta_3}E_3c^5 + F_3c^{-5} = \Delta u_{r4}(c), \end{aligned} \quad (111)$$

$$2\pi G_3c = ca, \quad (112)$$

$$\begin{aligned} & -C_2c^3 - D_2c^{-3} - E_2c^5 - F_2c^{-5} \\ & + C_3c^3 + D_3c^{-3} + E_3c^5 + F_3c^{-5} = \Delta u_{\theta4}(c). \end{aligned} \quad (113)$$

At $r=d$ Eq. (95) gives

$$\begin{aligned} & \frac{E_3}{1+\nu_3} [-A_3d^{-2} - G_3(1-\beta_3)(\frac{1}{2} + \frac{1}{\beta_3}\ln d) - (1-\beta_3)B_3 - \frac{1}{2}(2-\beta_3)\epsilon_{zz}] \\ & = -\frac{E_3}{1+\nu_3} \frac{1}{2}(4-3\beta_3)k_3, \end{aligned} \quad (114)$$

$$\frac{E_3}{1+v_3} \left[-3C_3c^2 - 9\frac{1-\beta_3}{2-\beta_3}D_3c^{-4} - 5\frac{1-\beta_3}{2-3\beta_3}E_3c^4 - 5F_3c^{-6} \right] = 0, \quad (115)$$

$$\frac{E_3}{1+v_3} \left[3C_3c^2 - 6\frac{1-\beta_3}{2-\beta_3}D_3c^{-4} + 10\frac{1-\beta_3}{2-3\beta_3}E_3c^4 - 5F_3c^{-6} \right] = 0. \quad (116)$$

Note that $\Delta\sigma_{rr0}$ and $\Delta\sigma_{rr4}$ are the isotropic and $\cos 4\theta$ terms of Eq. (82). Similarly $\Delta\sigma_{r\theta4}$ is the $\sin 4\theta$ term of Eq. (84). Likewise $\Delta u_{r0} = P_0$, $\Delta u_{r4} = P_4$ and $\Delta u_{\theta4} = Q_4$ of Eq. (75). Note also that the double current sheet excitation (Model A) is obtained by setting $\Delta\sigma_{rr} = \Delta\sigma_{\theta\theta} = \Delta\sigma_{r\theta} = \Delta u_r = \Delta u_\theta = 0$ while the thick cosine two theta excitation (Model B) is obtained by retaining the additional stresses and displacements but setting $f_{r0} = f_{r4} = f_{\theta4} = 0$.

Use of the Virial Theorem

The virial theorem is used to obtain the final relation among the unknowns. It states that

$$\iint (\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz}) r dr d\theta = \int_{r=r_s} \vec{r} \cdot \vec{\tau} \cdot \vec{n} r dr d\theta + W_B, \quad (117)$$

where the double integral is throughout the cross section of the material under stress. The single integral is over the cylinder at $r=r_s$ (iron shield) and W_B is the magnetic energy per unit length contained within the region bounded by $r=r_s$. The RHS of Eq. (117) has been evaluated in Eqs. (14) and (16) for Model A and in Eqs. (29-30) for Model B. From Eq. (47) in Ref. (2) one remembers that

$$\sigma_{zz} = v(\sigma_{rr} + \sigma_{\theta\theta}) + E(\varepsilon_{zz} - k). \quad (118)$$

The LHS of Eq. (117) becomes

$$\text{LHS}(117) = \iint [1+v](\sigma_{rr} + \sigma_{\theta\theta}) + E(\varepsilon_{zz} - k)] r dr d\theta, \quad (119)$$

or using Eqs. (52-53)

$$\begin{aligned} \text{LHS(117)} &= 2\pi f E(1-\beta) [-G(1+\frac{1}{\beta} + \frac{2}{\beta} \ln r) - \epsilon_{zz} + 3k - 2B] r dr \\ &\quad + 2\pi f (1+v) (\Delta\sigma_{rr} + \Delta\sigma_{\theta\theta}) r dr, \end{aligned} \quad (120)$$

where the subscripts have been dropped since it is to be summed over each structural region. The indefinite integral is

Indefinite LHS(117)

$$\begin{aligned} &= \pi E(1-\beta) [-G(1+\frac{2}{\beta} \ln r) - \epsilon_{zz} + 3k - 2B] r^2 \\ &\quad + \pi E \frac{\mu}{\beta} (1-\beta) [\frac{1}{4}(2+\lambda)r^4 - r^4 \ln r - b^4 \ln r], \end{aligned} \quad (121)$$

where Eqs. (82-83) were utilized to obtain the second term. Placing this second term on the RHS and summing over all regions gives the final relation.

$$\begin{aligned} &\pi E_1 (1-\beta_1) [-\epsilon_{zz} + 3k_1 - 2B_1] (b^2 - a^2) \\ &+ \pi E_2 (1-\beta_2) [-\epsilon_{zz} + 3k_2 - 2B_2] (c^2 - b^2) \\ &+ \pi E_3 (1-\beta_3) \left\{ [-G_3 - \epsilon_{zz} + 3k_3 - 2B_3] (d^2 - c^2) \right. \\ &\quad \left. - 2 \frac{G_3}{\beta_3} [d^2 \ln d - c^2 \ln c] \right\} = \text{RHS(122)}, \end{aligned} \quad (122)$$

where for Model A (double sheet cosine two theta)

$$\begin{aligned} \text{RHS(122)} &= \frac{1}{2}\pi^2 \left\{ b^2 (1+5b^4 r_s^{-4}) i_b^2 \right. \\ &\quad + 2 \frac{b^3}{c} (1+5c^4 r_s^{-4}) i_b i_c \\ &\quad \left. + c^2 (1+5c^4 r_s^{-4}) i_c^2 \right\}. \end{aligned} \quad (123)$$

In Eq. (123) i_b and i_c are given by Eqs. (19-21).

For Model B (thick cosine two theta)

$$\begin{aligned} \text{RHS(122)} &= \frac{1}{16}\pi^2 J_o^2 \left\{ -4b^4 \ln \frac{c}{b} + (c^4 - b^4) [1 + \frac{5}{2}(c^4 - b^4) r_s^{-4}] \right\} \\ &\quad + \frac{1}{8}\pi^2 J_o^2 \frac{1+\nu}{1-\nu} \frac{2}{2} \left\{ -4b^4 \ln \frac{c}{b} + (c^4 - b^4) [1 + \frac{1}{2}(c^4 - b^4) r_s^{-4}] \right\}, \end{aligned} \quad (124)$$

where J_o is given in Eq. (31).

Numerical Calculations

The condition of stress, strain and displacement that exists in three nested hollow cylinders as a result of thermal cooldown, pretension in the outer cylinder, and two different cosine two theta axial current distributions in the middle cylinder has been calculated as a function of the central quadrupole gradient. Twenty algebraic relations in Eqs.(98-116) and Eq. (122) among the twenty unknown coefficients ($A_1 B_1 C_1 D_1 E_1 F_1 A_2 B_2 C_2 D_2 E_2 F_2 A_3 G_3 B_3 C_3 D_3 E_3 F_3 \varepsilon_{zz}$) have been solved. Thus, for example, the state of stress at any point in the quadrupole structure may be found. It is usually clear whether a quantity is stress or strain. Otherwise, R is radial, T is theta or azimuthal, Z is axial or longitudinal. With regard to position A,B,C,D are the radii bounding the various media. To indicate which side of a boundary radius, P is used for positive and M for negative. Thus, for example, RTBP indicates the (r, θ) component at radius B but in the material between B and C. As explained previously⁽²⁾ $\sqrt{3J_2}$ is a stress which is to be compared with the yield stress in tension.

Numerical results are given relative to a beam line quadrupole. The following cases were calculated

| Case | Cool Down | Pretension | Excitation |
|------|-----------|------------|------------|
| 1 | No | No | No |
| 2 | No | No | Yes |
| 3 | No | Yes | No |
| 4 | No | Yes | Yes |
| 5 | Yes | No | No |
| 6 | Yes | No | Yes |
| 7 | Yes | Yes | No |
| 8 | Yes | Yes | Yes |

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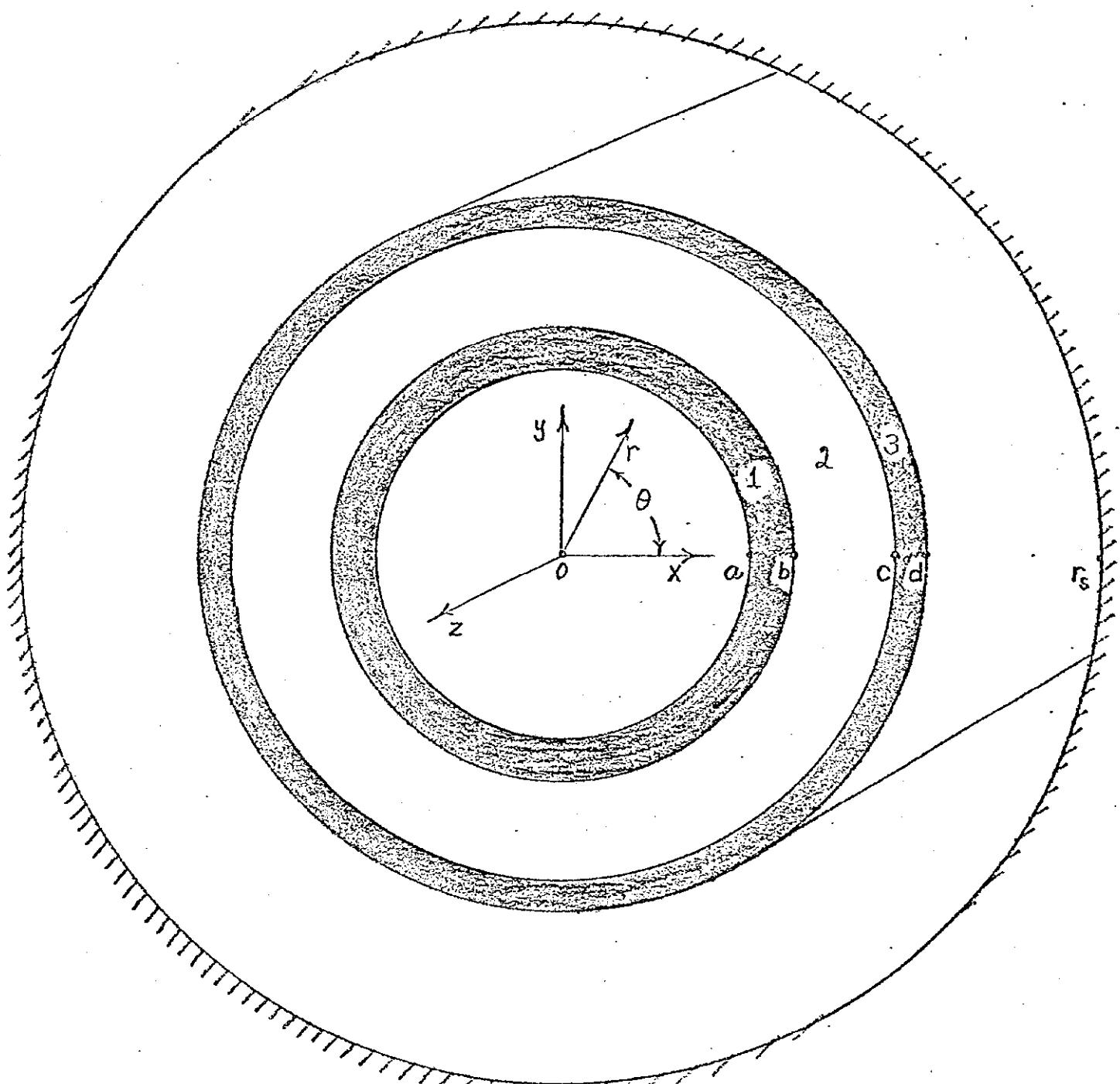


Fig. 1. Geometric Details of Doubler Dipole Model

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE EQUAD

CENTRAL GRADIENT (KG/IN) = 0.0000 INNER BORE TUBE RADIUS(IN) = 3.0000 INNER CONDUCTOR RADIUS(IN) = 3.5000
 OUTER CONDUCTOR RADIUS(LBS/IN) = 5.6130 OUTER RADIUS OF IRON SHIELD(LBS/IN) = 6.0000 BAND YOUNG'S MODULUS(LBS/IN) = 1000000 BAND YOUNG'S MODULUS(MN/M) = 11000000
 COND. POISSON'S RATIO(1/2) = 0.3333 COND. POISSON'S RATIO(MN/M) = 0.3333
 COND. THERMAL EXPANSION(CIN/CIN) = 0.0003 COND. THERMAL ENERGY(J/M) = 0.00 MAGNETIC ENERGY(CIN-LBS)/IN) = 0.00

| ANGLE (DEG) | RRAP | TTAP | RRBM | TTBM | RRCP | TTCP | RRDM | TTDM | RTDM |
|-------------|------|------|------|------|------|------|------|------|------|
| 45.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 33.75 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 22.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 11.25 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

- 25 -

TRANSVERSE STRAIN (MUN/IN) RTSP TTSB RTSP TTSB

| ANGLE (DEG) | RRAP | TTAP | RRBM | TTBM | RRCP | TTCP | RRDM | TTDM | RTDM |
|-------------|------|------|------|------|------|------|------|------|------|
| 45.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 33.75 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 22.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 11.25 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

- 26 -

TRANSVERSE DISPLACEMENT (IN) UTA URB UTC URC UTB UTD UTO

| ANGLE (DEG) | ZZAP | ZZBM | ZZCM | ZZDM | ZZEM | ZZFM | ZZGM | ZZHM | ZZIM | ZZJM | ZZKM | ZZLM | ZZMM | ZZNM | ZZPM | ZZQM | ZZRM | ZZSM | ZZTM | ZZUM | ZZVM |
|-------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 45.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| 33.75 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| 22.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| 11.25 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

Case 1. No Cool Down, No Pre Load, No Excitation

CENTRAL GRADIENT (KG/IN)
 CENTER CONDUCTOR RADIUS (IN) = 14.9263 INNER BORE TUBE RADIUS (IN) = 3.0000 INNER CONDUCTOR RADIUS (IN)
 CENTER CONDUCTOR RADIUS (IN) = 15.3130 OUTER RADIUS OF BANDS (IN) = 6.5000 RADIUS OF IRON SHIELD (IN)
 CENTER POISSON'S RATIO (0.5FSIN/IN) = 29000.000 BAND YOUNG'S MODULUS (LSIN/IN) = 100000.000 BAND YOUNG'S MODULUS (LSIN/IN)
 CENTER STRAIN (IN/IN) = 0.0000 BAND POLYMER STRAIN (IN/IN) = 0.0000 BAND POLYMER STRAIN (IN/IN)
 CENTER STRESS (MILLIBAR) = 0.0000 BAND POLYMER STRAIN (IN/IN) = 0.0000 BAND POLYMER STRAIN (IN/IN)
 TYPE OF COMPUTATION = ELOUAS MAGNETIC ENERGY (J/IN) = 127140.73

| ANGLE (DEG) | R2AP | TTAP | RRBM | TTBM | RRBM | TTBP | RRBP | TTCP | RRCP | TTDM | RRDM | RTOP | |
|-------------|-----------|------|--------|---------|---------|--------|------|---------|---------|--------|------|---------|-------|
| | | | | | | | | | | | | | |
| 45.0 | -118.04. | -0. | 24.3 | 111.10. | 9. | 24.3 | 43. | 193. | 275. | 6. | 193. | 401.2. | -989. |
| 45.0 | -117.946. | 0. | 27.1 | 117.14. | 115.6. | 27.1 | 51. | 164. | 258. | 179. | 184. | 378.3. | 0. |
| 35.0 | -32.61. | 0. | 34.6. | 95.674. | 12.3. | 34.6. | 73. | 1166.1. | 1167. | 136.6. | 156. | 314.6. | 0. |
| 25.0 | -22.61. | 0. | 46.9. | 73.23. | 116.61. | 46.9. | 196. | 1166.1. | 1167. | 135. | 152. | 215.1. | 0. |
| 15.0 | 0. | 0. | 56.36. | 61.7. | 119.69. | 61.7. | 147. | 1166.9. | 1167. | 135. | 152. | 545.5. | 0. |
| 5.0 | 82.04. | 0. | 67.4. | 52.64. | 119.69. | 67.4. | 74. | 1188.9. | 1189. | 151. | 152. | 545.5. | 0. |
| -5.0 | 137.63. | 0. | 92.1. | 12.64. | 116.61. | 92.1. | 274. | 1166.1. | 1167. | 166. | 153. | 133.4. | 0. |
| -15.0 | 182.98. | 0. | 194.1. | 148.3. | 112.35. | 194.1. | 268. | 112.35. | 112.35. | 162. | 153. | 153.0. | 0. |
| -25.0 | 212.58. | 0. | 112.6. | 114.7. | 116.61. | 112.6. | 285. | 116.61. | 1167. | 123. | 152. | 152.8. | 0. |
| -35.0 | 322.68. | 0. | 114.7. | 114.7. | 114.7. | 294. | 294. | 114.7. | 114.7. | 125. | 152. | 1317.1. | 0. |
| -45.0 | 432.78. | 0. | 114.7. | 114.7. | 114.7. | 307. | 307. | 114.7. | 114.7. | 125. | 152. | 339.4. | 0. |

| ANGLE (DEG) | R2AP | TTAP | RRBM | TTBM | RRBM | TTBP | RRBP | TTCP | RRCP | TTDM | RRDM | RTOP | TRANSVERSE STRAIN (L/IN/IN) |
|-------------|--------|---------|--------|--------|------|--------|--------|--------|-------|--------|------|------|-----------------------------|
| | | | | | | | | | | | | | |
| 45.0 | 12.6. | -417. | -0. | -217. | 292. | 0. | 220. | -0. | 239. | 277. | -9. | 232. | 259. |
| 35.0 | -14.5. | -33.5. | 0. | -211. | 273. | -37. | 249. | 4. | 222. | -123. | 0. | 123. | 0. |
| 25.0 | -15.5. | -12.94. | 0. | -16.0. | 171. | -57. | 321. | 17. | 135. | -194. | 0. | 194. | 0. |
| 15.0 | -19. | -11.15. | 0. | -12.3. | 179. | -87. | 45.7. | 57. | 143. | -24. | 0. | 24. | 0. |
| -5.0 | -13.5. | -19.7. | 0. | -16.7. | 149. | -87. | 67. | 120.7. | -82. | 193. | -93. | 49. | 0. |
| -15.0 | -33.5. | -36.7. | 0. | -4.6. | 136. | -76. | 87. | 267.3. | -15. | 25. | -82. | 75. | 0. |
| -25.0 | -53.5. | -58.6. | 0. | -1.4. | 136. | -76. | 97.5. | 111. | 182. | -13. | 14. | 14. | 0. |
| -35.0 | -73.6. | -62.8. | 0. | -1.4. | 134. | -76. | 106.7. | 144. | 172. | -31.5. | 14. | 14. | 0. |
| -45.0 | -93.6. | -62.8. | 0. | -1.4. | 134. | -76. | 106.7. | 144. | 172. | -31.5. | 14. | 14. | 0. |
| -5.0 | 12.6. | 77.08. | 0. | -217. | 173. | 3082. | 1431. | 165. | 12.2. | 9.8. | 2.2. | 3.7. | 0. |
| 15.0 | 34.2. | 85.63. | 193. | -172. | 173. | 3911. | 1551. | 165. | 11.4. | 9.8. | 1.2. | 5. | 0.005. |
| 25.0 | 45.0. | 118.4. | 84.35. | -12.7. | 173. | 22856. | 11580. | 165. | 11.4. | 8.8. | 1.2. | 3.2. | 0.017. |
| 35.0 | 56.0. | 124.7. | 99.77. | -22.2. | 173. | 2492. | 1492. | 165. | 6.6. | 7.6. | 3.9. | 2.6. | 0.031. |
| 45.0 | 67.0. | 79.50. | 79.32. | -22.2. | 152. | 2441. | 152. | 165. | 4.8. | 6.6. | 3.9. | 2.6. | 0.051. |
| 55.0 | 78.0. | 85.92. | 75.03. | -22.2. | 152. | 2616. | 152. | 165. | 4.7. | 5.7. | 3.9. | 2.6. | 0.071. |
| 65.0 | 89.6. | 103.56. | 95.6. | -22.2. | 152. | 2809. | 152. | 165. | 4.7. | 5.3. | 3.9. | 2.6. | 0.092. |
| 75.0 | 100.0. | 118.51. | 46.05. | -22.2. | 152. | 2322. | 146. | 165. | 4.7. | 5.5. | 1.4. | 2.6. | 0.112. |
| 85.0 | 110.0. | 121.94. | 44.62. | -22.2. | 152. | 2322. | 146. | 165. | 4.7. | 5.6. | .8. | 2.6. | 0.132. |

ANGLE (DEG) LONG. STRESS (LB/IN/IN) AND STRAIN (MLN/IN/IN) SRT (3*J2) IN YAP Y34 YBP YCP YCN YDN YUR UTD

TRANSVERSE DISPLACEMENT (IN) UTA UTP UTC URC UTC UTD

Case 2. No Cool Down, No Pre Load, With Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE ELQUAD

CENTRAL GRADIENT (KG/IN) = 0.9000 INNER TUBE RADIUS (IN) = 3.0000 INNER CONDUCTOR RADIUS (IN) = 3.5000
 CENTER CONDUCTOR RADIUS (IN) = 5.8135 CYLINDRICAL RADIUS OF BANDING (IN) = 6.5000 RADIUS OF IRON SHIELD (IN) = 9.0000
 CORE YOUNG'S MODULUS (LBS/IN/IN) = 29000000 COND. YOUNG'S MODULUS (LBS/IN/IN) = 10000000 BAND POISSON'S RATIO (.000001) = 0.00000001
 CORE POISSON'S RATIO (.000001) = 0.3333 BAND THESSALON STRAIN (IN/IN) = 0.0000 BAND THESSALON STRAIN (IN/IN) = 0.0000 MAGNETIC ENERGY (J/1A) = 0.0000
 BAND DISLOCATION (MILLIRAD) = 2.500 MAGNETIC ENERGY (J/1A) = 0.0000 MAGNETIC ENERGY (J/1A) = 0.0000
 TYPE OF COMPUTATION = ELQUAS

| ANGLE (DEG) | RRAP | TTAP | RTBM | RRBM | TTBM | RRBP | TTBP | TRANSVERSE STRESS (LB/IN/IN) | | RTCM | RRCP | TTCP | RRDM | TTDM | RTDN |
|----------------|------|---------|------|--------|---------|------|--------|------------------------------|------|-------|------|-------|--------|------|--------|
| | | | | | | | | FROM | FROM | | | | | | |
| 45.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 45.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 35.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 35.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 25.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 25.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 15.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 15.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 15.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 15.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |
| 0.0 | 0. | -30.89. | 0. | -41.0. | -267.9. | 0. | -41.0. | -119. | 0. | -212. | 0. | -217. | 25.65. | -0. | 27.98. |

| ANGLE (DEG) | RRAP | TTAP | RTBM | RRBM | TTBM | RRBP | TTBP | TRANSVERSE STRAIN (MUIN/IN/IN) | | RTCM | RRCP | TTCP | RRDM | TTDM | RTDN |
|----------------|------|---|--------|--------------------------|------|------------------------------|-------|--------------------------------|------|-----------------------------|-------|-------------------------|------|-----------------------------|-------|
| | | | | | | | | FROM | FROM | | | | | | |
| 45.0 | 0. | 52. | -90. | 0. | 33. | -72. | 0. | -394. | -72. | 0. | -289. | -174. | 0. | -125. | 23.9. |
| 45.0 | 0. | 52. | -90. | 0. | 33. | -72. | 0. | -394. | -72. | 0. | -289. | -174. | 0. | -125. | 23.9. |
| 35.0 | 0. | 52. | -90. | 0. | 33. | -72. | 0. | -394. | -72. | 0. | -289. | -174. | 0. | -125. | 23.9. |
| 35.0 | 0. | 52. | -90. | 0. | 33. | -72. | 0. | -394. | -72. | 0. | -289. | -174. | 0. | -125. | 23.9. |
| 25.0 | 0. | 52. | -90. | 0. | 33. | -72. | 0. | -394. | -72. | 0. | -289. | -174. | 0. | -125. | 23.9. |
| 25.0 | 0. | 52. | -90. | 0. | 33. | -72. | 0. | -394. | -72. | 0. | -289. | -174. | 0. | -125. | 23.9. |
| 15.0 | 0. | 52. | -90. | 0. | 33. | -72. | 0. | -394. | -72. | 0. | -289. | -174. | 0. | -125. | 23.9. |
| 15.0 | 0. | 52. | -90. | 0. | 33. | -72. | 0. | -394. | -72. | 0. | -289. | -174. | 0. | -125. | 23.9. |
| 0.0 | 0. | 52. | -90. | 0. | 33. | -72. | 0. | -394. | -72. | 0. | -289. | -174. | 0. | -125. | 23.9. |
| ANGLE (DEG) | | LONG. STRESS (LB/IN/IN) AND STRAIN (MUIN/IN/IN) | | SQR(3*J2)*IN (KLP/IN/IN) | | TRANSVERSE DISPLACEMENT (IN) | | YAP YBM YCP YCK YCR YDR YTR | | TRANVERSE DISPLACEMENT (IN) | | YAP YBM YCP YCR YDR YTR | | TRANVERSE DISPLACEMENT (IN) | |
| 45.0 | 0. | 1397. | -1397. | -66. | -66. | 609. | -792. | -13. | -2.7 | 2.9 | -3. | 2.5 | -2.5 | 2.5 | 2.5 |
| 45.0 | 0. | 1397. | -1397. | -66. | -66. | 609. | -792. | -13. | -2.7 | 2.9 | -3. | 2.5 | -2.5 | 2.5 | 2.5 |
| 35.0 | 0. | 1397. | -1397. | -66. | -66. | 609. | -792. | -13. | -2.7 | 2.9 | -3. | 2.5 | -2.5 | 2.5 | 2.5 |
| 35.0 | 0. | 1397. | -1397. | -66. | -66. | 609. | -792. | -13. | -2.7 | 2.9 | -3. | 2.5 | -2.5 | 2.5 | 2.5 |
| 25.0 | 0. | 1397. | -1397. | -66. | -66. | 609. | -792. | -13. | -2.7 | 2.9 | -3. | 2.5 | -2.5 | 2.5 | 2.5 |
| 25.0 | 0. | 1397. | -1397. | -66. | -66. | 609. | -792. | -13. | -2.7 | 2.9 | -3. | 2.5 | -2.5 | 2.5 | 2.5 |
| 15.0 | 0. | 1397. | -1397. | -66. | -66. | 609. | -792. | -13. | -2.7 | 2.9 | -3. | 2.5 | -2.5 | 2.5 | 2.5 |
| 15.0 | 0. | 1397. | -1397. | -66. | -66. | 609. | -792. | -13. | -2.7 | 2.9 | -3. | 2.5 | -2.5 | 2.5 | 2.5 |
| 0.0 | 0. | 1397. | -1397. | -66. | -66. | 609. | -792. | -13. | -2.7 | 2.9 | -3. | 2.5 | -2.5 | 2.5 | 2.5 |

Case 3. No Cool Down, With Pre Load, No Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE ELQUAD

CENTRAL GRADIENT (KG/IN) = 14.8000 INNER BOBE TUBE RADIUS (IN) = 3.0000 INNER CONDUCTOR RADIUS (IN) = 3.5000
 OUTER CONDUCTOR RADIUS (IN) = 5.8130 OUTER RADIUS OF BANDING (IN) = 6.5000 RADIUS OF IRON SHIELD (IN) = 6.5000
 BOBE YOUNG'S MODULUS (LBS/IN/IN) = 29.00000 COND. YOUNG'S MODULUS (LBS/IN/IN) = 10.00000 BAND YOUNG'S MODULUS (LBS/IN/IN) = 11.00000
 BOBE POISSON'S STRAIN (IN/IN) = 0.3000 COND. POISSON'S STRAIN (IN/IN) = 0.3000 BAND POISSON'S STRAIN (IN/IN) = 0.3000
 BAND DISLOCATIONAL ENERGY (IN-LBS)/IN = 0.2500 MAGNETIC ENERGY (J/M) = 127140.73 MAGNETIC ENERGY (J/M) = 566354.17
 TYPE OF COMPUTATION = ELQUAS

| ANGLE (DEG) | RRAP | | | RTAP | | | RFBM | | | TTBM | | | TRANSVERSE STRESS (LB/IN/IN) | | | |
|----------------|------|---------|------|--------|--------|--------|--------|--------|--------|--------|------|-------|------------------------------|--------|---------|---------|
| | RTPA | RTPB | RTPC | RTPA | RTPB | RTPC | RFBM | RFBM | RFBM | TTBM | TTBM | TTBM | RRCP | RRCP | RRDM | |
| 45.0 | -0.0 | -146.92 | -0.0 | -166. | 64.31 | 0.0 | -166. | -76. | 0.0 | -510. | 63. | 0.0 | -510. | 657.8 | -1.0 | -1.819. |
| 45.0 | -0.0 | -138.64 | -0.0 | -139. | 80.35 | 0.0 | -139. | -68. | -656. | -150. | 45. | -170. | -531. | -1.0 | -1.837. | |
| 35.0 | -0.0 | -129.95 | -0.0 | -91. | 68.95 | -1.253 | -1253. | -1253. | -1253. | -1473. | 336. | -136. | -136. | -213.6 | -0.0 | -0.0 |
| 35.0 | -0.0 | -123.70 | -0.0 | -60. | 51.49 | -1.156 | -1156. | -1156. | -1156. | -1473. | 345. | -145. | -145. | -244.5 | -0.0 | -0.0 |
| 25.0 | -0.0 | -116.07 | -0.0 | -60. | 3.97 | -1.062 | -1062. | -1062. | -1062. | -1473. | 354. | -154. | -154. | -282.9 | -0.0 | -0.0 |
| 25.0 | -0.0 | -107. | -0.0 | -207. | 3.97 | -1.062 | -1062. | -1062. | -1062. | -1473. | 354. | -154. | -154. | -282.9 | -0.0 | -0.0 |
| 15.0 | -0.0 | -51.12 | -0.0 | -264. | 2.64 | -1.062 | -1062. | -1062. | -1062. | -1473. | 364. | -164. | -164. | -374.2 | -0.0 | -0.0 |
| 15.0 | -0.0 | -136.75 | -0.0 | -51.12 | 2.64 | -1.145 | -1145. | -1145. | -1145. | -1473. | 374. | -174. | -174. | -415. | -0.0 | -0.0 |
| 15.0 | -0.0 | -152.09 | -0.0 | -632. | 1.23 | -1.23 | -123. | -123. | -123. | -1473. | 384. | -184. | -184. | -454. | -0.0 | -0.0 |
| 15.0 | -0.0 | -181.69 | -0.0 | -716. | -3.12 | -1.23 | -123. | -123. | -123. | -1473. | 394. | -194. | -194. | -494. | -0.0 | -0.0 |
| 0.0 | -0.0 | -191.97 | -0.0 | -737. | -4.698 | -0.0 | -737. | -175. | -0.0 | -193. | -5. | -823. | -823. | -435. | -0.0 | -0.0 |

| ANGLE (DEG) | RRAP | | | RTAP | | | RFBM | | | TTBM | | | TRANSVERSE STRAIN (MU/IN/IN) | | | DISPLACEMENT (IN) | | | |
|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|------------------------------|-------|------|-------------------|--------|------|-----|
| | RTPA | RTPB | RTPC | RTPA | RTPB | RTPC | RFBM | RFBM | RFBM | TTBM | TTBM | TTBM | RRCP | RRCP | RRDM | RTDP | RTDP | RTDM | |
| 45.0 | 177. | -5.97 | -0.0 | -195. | 210. | 0.0 | -171. | -72. | 0.0 | -528. | 104. | -104. | -157. | 592. | -2.0 | -1.24. | 96. | 0.0 | |
| 45.0 | 162. | -4.76 | -0.0 | -178. | 198. | -3.0 | -145. | -72. | -72. | -516. | 165. | -165. | -134. | 483. | -2.0 | -1.20. | 102. | 0.0 | |
| 35.0 | 116. | -3.85 | -0.0 | -158. | 162. | -2.7 | -120. | -55. | -136. | -136. | 325. | -325. | -319. | 439. | -2.0 | -1.36. | 114. | 0.0 | |
| 35.0 | 47. | -2.46 | -0.0 | -120. | 162. | -1.25 | -105. | -45. | -127. | -127. | 325. | -325. | -319. | 349. | -2.0 | -1.47. | 114. | 0.0 | |
| 25.0 | -38. | -1.75 | -0.0 | -129. | 175. | -0.90 | -97. | -47. | -127. | -127. | 325. | -325. | -319. | 359. | -2.0 | -1.58. | 114. | 0.0 | |
| 25.0 | -129. | -1.96 | -0.0 | -124. | 177. | -0.51 | -104. | -51. | -127. | -127. | 325. | -325. | -319. | 369. | -2.0 | -1.69. | 114. | 0.0 | |
| 15.0 | -124. | -2.17 | -0.0 | -113. | 177. | -0.13 | -102. | -57. | -127. | -127. | 325. | -325. | -319. | 379. | -2.0 | -1.80. | 114. | 0.0 | |
| 15.0 | -124. | -2.37 | -0.0 | -416. | 177. | -0.17 | -117. | -57. | -127. | -127. | 325. | -325. | -319. | 389. | -2.0 | -1.91. | 114. | 0.0 | |
| 15.0 | -324. | 5.96 | -0.0 | -129. | 177. | -0.51 | -129. | -66. | -66. | -127. | 325. | -325. | -319. | 399. | -2.0 | -1.92. | 114. | 0.0 | |
| 0.0 | -344. | 5.16 | -0.0 | -47. | 6.0 | -0.4 | -206. | -72. | -72. | -127. | 325. | -325. | -319. | 409. | -2.0 | -1.93. | 114. | 0.0 | |
| 45.0 | 0.0 | 162. | -4.76 | -0.0 | -178. | 198. | -3.0 | -145. | -72. | -72. | -516. | 165. | -165. | -134. | 483. | -2.0 | -1.24. | 96. | 0.0 |
| 45.0 | 0.0 | 116. | -3.85 | -0.0 | -158. | 162. | -2.7 | -120. | -55. | -136. | -136. | -136. | -136. | -136. | -2.0 | -1.20. | 102. | 0.0 | |
| 35.0 | 0.0 | 47. | -2.46 | -0.0 | -120. | 162. | -1.25 | -105. | -45. | -127. | -127. | -127. | -127. | -127. | -2.0 | -1.36. | 114. | 0.0 | |
| 35.0 | 0.0 | -38. | -1.75 | -0.0 | -129. | 175. | -0.90 | -97. | -47. | -127. | -127. | -127. | -127. | -127. | -2.0 | -1.47. | 114. | 0.0 | |
| 25.0 | 0.0 | -129. | -1.96 | -0.0 | -124. | 177. | -0.51 | -104. | -51. | -127. | -127. | -127. | -127. | -127. | -2.0 | -1.58. | 114. | 0.0 | |
| 25.0 | 0.0 | -124. | -2.17 | -0.0 | -416. | 177. | -0.13 | -117. | -57. | -127. | -127. | -127. | -127. | -127. | -2.0 | -1.69. | 114. | 0.0 | |
| 15.0 | 0.0 | -124. | -2.37 | -0.0 | -416. | 177. | -0.17 | -117. | -57. | -127. | -127. | -127. | -127. | -127. | -2.0 | -1.80. | 114. | 0.0 | |
| 15.0 | 0.0 | -344. | 5.96 | -0.0 | -47. | 6.0 | -0.4 | -206. | -72. | -72. | -127. | 325. | -325. | -319. | 409. | -2.0 | -1.91. | 114. | 0.0 |
| 0.0 | 0.0 | -344. | 5.16 | -0.0 | -47. | 6.0 | -0.4 | -206. | -72. | -72. | -127. | 325. | -325. | -319. | 409. | -2.0 | -1.92. | 114. | 0.0 |

Case 4. No Cool Down, With Pre Load, With Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE E1QUAD

CENTRAL GRADIENT (KG/IN) = 0.0000 INNER POLE RADIUS (IN) = 3.6000 INNER CONDUCTOR RADIUS (IN) = 3.5000
 OUTER CONDUCTOR RADIUS (IN) = 5.013 OUTTER RADIUS OF BANDING (IN) = 6.5000 RADIIUS OF TORSION SHIFTS (IN)
 POLE MODULUS (LBS/IN/IN) = 2900.000 YOUNG'S MODULUS (LBS/IN/IN) = 1000.000 RADIIUS OF POISSON'S RATIO (IN/IN)
 BORE POISSON'S RATIO = .3333 COND. RADIUS (IN/IN) = 1.0000 BAND POISSON'S RATIO (IN/IN) = 1.0000
 BORE THERMAL STRAIN (IN/IN) = -.0033 COND. THERMAL STRAIN (IN/IN) = .0033 BAND THERMAL STRAIN (IN/IN) = .0033
 BORE DISLOCATION (MILLIPAO) = .0015 MAGNETIC ENERGY (J/IN-LAS) = 0.0000 MAGNETIC ENERGY (J/IN-LAS) = 0.0000
 TYPE OF COMPUTATION = ELOQUAD

| ANGLE (DEG) | RRAP | TTAP | RTAP | RRBM | TTBM | RTBM | TRANSVERSE STRESS (LBS/IN/IN) | | RRCP | TTCP | RTCP | RRDM | TTDM | RTDM | |
|----------------|------|------|---------|---------|--------|--------|-------------------------------|--------|--------|--------|------|--------|--------|------|---|
| | | | | | | | RRCP | TTCP | | | | | | | |
| 4.5 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 4.5 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 3.5 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 3.5 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 2.5 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 2.5 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 1.5 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 1.5 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 1.5 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 0 | 0 | 0 | -109.35 | 0 | -14.51 | 94.84 | 0 | -1451. | -220. | 0 | -94. | -476. | 0 | 0 | |
| 4.5 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 4.5 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 3.5 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 3.5 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 2.5 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 2.5 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 1.5 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 1.5 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 1.5 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 0 | 0 | 0 | -264.2 | -31.44 | 0 | -27.08 | 13.078 | 0 | -4652. | -3078. | 0 | -4150. | -3579. | 0 | 0 |
| 4.5 | 0 | 0 | 724P | E1QUAD | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4.5 | 0 | 0 | 724H | ZZCH | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4.5 | 0 | 0 | 20273 | -20273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 4.5 | 0 | 0 | 20273 | -20273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 3.5 | 0 | 0 | 20273 | -20273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 3.5 | 0 | 0 | 20273 | -20273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 2.5 | 0 | 0 | 20273 | -20273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 2.5 | 0 | 0 | 20273 | -20273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 1.5 | 0 | 0 | 20273 | -20273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 1.5 | 0 | 0 | 20273 | -20273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 0 | 0 | 0 | 20273 | -20273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 4.5 | 0 | 0 | 290273 | -290273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 4.5 | 0 | 0 | 290273 | -290273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 3.5 | 0 | 0 | 290273 | -290273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 3.5 | 0 | 0 | 290273 | -290273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 2.5 | 0 | 0 | 290273 | -290273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 2.5 | 0 | 0 | 290273 | -290273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 1.5 | 0 | 0 | 290273 | -290273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 1.5 | 0 | 0 | 290273 | -290273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |
| 0 | 0 | 0 | 290273 | -290273 | -471. | -471. | 0 | 8987. | 8987. | 17.6 | 16.4 | 1.3 | 0 | 0 | 0 |

ANGLE LONG. STRESS (LB/IN²/IN/IN) AND STRAIN (MU/IN/IN)
 (DEG) 724P TRANSVERSE DISPLACEMENT (IN)
 4.5 0 YB1 YB2 YC1 YC2 YM1 YM2 UTD UTD UTD UTD UTD UTD UTD UTD UTD
 4.5 0 20273 -20273 -471. -471. 0 8987. 8987. 17.6 16.4 1.3 0 0 0 0 0 0 0 0 0
 4.5 0 20273 -20273 -471. -471. 0 8987. 8987. 17.6 16.4 1.3 0 0 0 0 0 0 0 0 0
 3.5 0 20273 -20273 -471. -471. 0 8987. 8987. 17.6 16.4 1.3 0 0 0 0 0 0 0 0 0
 3.5 0 20273 -20273 -471. -471. 0 8987. 8987. 17.6 16.4 1.3 0 0 0 0 0 0 0 0 0
 2.5 0 20273 -20273 -471. -471. 0 8987. 8987. 17.6 16.4 1.3 0 0 0 0 0 0 0 0 0
 2.5 0 20273 -20273 -471. -471. 0 8987. 8987. 17.6 16.4 1.3 0 0 0 0 0 0 0 0 0
 1.5 0 20273 -20273 -471. -471. 0 8987. 8987. 17.6 16.4 1.3 0 0 0 0 0 0 0 0 0
 1.5 0 20273 -20273 -471. -471. 0 8987. 8987. 17.6 16.4 1.3 0 0 0 0 0 0 0 0 0
 0 0 0 20273 -20273 -471. -471. 0 8987. 8987. 17.6 16.4 1.3 0 0 0 0 0 0 0 0 0

Case 5. Cool Down, No Pre Load, No Excitation

Case 6. Cool Down, No Pre Load, No Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE

CENTRAL CONDUCTOR RADIUS (IN) = 14.8000 INNER TUBE RADIUS (IN) = 3.3999 INNER CONDUCTOR RADIUS (IN) = 3.5000
 CENTER CONDUCTOR RADIUS (IN) = 14.8130 OUTER TUBE RADIUS (IN) = 6.5000 RADIUS OF IRON SHIELD (IN) = 6.5000
 BORE YOUNG'S MODULUS (LBS/IN/IN) = 29000.000 COND. YOUNG'S MODULUS (LBS/IN/IN) = 1000.000 RATIO OF IRON YOUNG'S MODULUS (IN/IN) = 1.1000
 BORE POISSONS RATIO = 0.333 COND. POISSONS RATIO = 0.333 BAND POISSONS RATIO (IN/IN) = 1.0000
 BORE THERMAL STRAIN (IN/IN) = -0.3000 COND. THERMAL STRAIN (IN/IN) = 1.3033 BAND THERMAL STRAIN (IN/IN) = 1.27140.73
 BORE DISLOCATON (IN/LIN/RAD) = 0.3000 MAGNETIC ENERGY (J/JR) = 566354.17
 TYPE OF COMPUTATION = ELQUAS

| ANGLE (DEG) | RRAP | | TTAP | | RRBM | | TTBM | | TRANVERSE STRESS (LB/IN/IN) FRCN | | RTCM | | RRCP | | TTCP | | RTDM | | TTDM | |
|------------------------|-------------|-------------|-------------|-------------|---|-------------|-------------|-------------|----------------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------|----|
| | RRAp | TTAp | RRTap | TTTAp | RRBm | TTBm | RRTbm | TTTbm | RRCp | TTcp | RTcm | RTdm | RRTdm | TTTdm | RRTcp | TTcp | RTdm | TTdm | | |
| 4.5° | 0. | -22739. | -0. | -1297. | 1626. | 0. | -11207. | 24. | -1183. | -204. | -1186. | -12952. | -170. | -1183. | -5957. | 0. | 0. | -1183. | 0. | |
| 4.0° | 0. | -21711. | -0. | -11180. | 1230. | -656. | -11205. | 31. | -1178. | -214. | -179. | -12729. | -170. | -1273. | -7074. | 0. | 0. | -1273. | 0. | |
| 3.5° | 0. | -15752. | -0. | -11031. | 1290. | -1232. | -11041. | 53. | -1233. | -226. | -1236. | -12395. | -170. | -1236. | -7253. | 0. | 0. | -1236. | 0. | |
| 3.0° | 0. | -14216. | -0. | -11661. | 1656. | -1261. | -11671. | 36. | -1154. | -214. | -1155. | -11168. | -170. | -1155. | -7589. | 0. | 0. | -1155. | 0. | |
| 2.5° | 0. | -13654. | -0. | -1634. | 1689. | -1284. | -1634. | 36. | -1689. | -216. | -1686. | -1551. | -170. | -1686. | -8731. | 0. | 0. | -1686. | 0. | |
| 2.0° | 0. | -2734. | -0. | -677. | 6978. | -14989. | -677. | 1271. | -1689. | -1604. | -1604. | -1605. | -170. | -1604. | -9439. | 0. | 0. | -1604. | 0. | |
| 1.5° | 0. | -2829. | -0. | -529. | 6927. | -821. | -529. | 215. | -1621. | -1621. | -1621. | -1621. | -170. | -1621. | -8457. | 0. | 0. | -1621. | 0. | |
| 1.0° | 0. | -7363. | -0. | -439. | 9967. | -1233. | -439. | 215. | -1631. | -1631. | -1631. | -1631. | -170. | -1631. | -8657. | 0. | 0. | -1631. | 0. | |
| 0.5° | 0. | -10323. | -0. | -331. | 11107. | -6556. | -331. | 266. | -1626. | -1626. | -1626. | -1626. | -170. | -1626. | -9157. | 0. | 0. | -1626. | 0. | |
| 0. | 0. | -11351. | -0. | -304. | -11503. | -0. | -304. | 0. | -304. | -0. | -676. | -0. | -676. | -0. | -676. | -5545. | -3. | 0. | -676. | 0. |
| ANGLE (DEG) | RRAP | TTAP | RRBM | TTBM | TRANVERSE STRESS (LB/IN/IN) FRCN | RTCM | RRCP | TTCP | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | | |
| 4.5° | 0. | -22739. | -0. | -1297. | 1626. | 0. | -11207. | 24. | -1183. | -204. | -1186. | -12952. | -170. | -1183. | -5957. | 0. | 0. | -1183. | 0. | |
| 4.0° | 0. | -21711. | -0. | -11180. | 1230. | -656. | -11205. | 31. | -1178. | -214. | -179. | -12729. | -170. | -1273. | -7074. | 0. | 0. | -1273. | 0. | |
| 3.5° | 0. | -15752. | -0. | -11031. | 1290. | -1232. | -11041. | 53. | -1233. | -226. | -1236. | -12395. | -170. | -1236. | -7253. | 0. | 0. | -1236. | 0. | |
| 3.0° | 0. | -14216. | -0. | -11661. | 1656. | -1261. | -11671. | 36. | -1154. | -214. | -1155. | -11168. | -170. | -1155. | -7589. | 0. | 0. | -1155. | 0. | |
| 2.5° | 0. | -13654. | -0. | -1634. | 1689. | -1284. | -1634. | 36. | -1689. | -216. | -1686. | -1551. | -170. | -1686. | -8731. | 0. | 0. | -1686. | 0. | |
| 2.0° | 0. | -2734. | -0. | -677. | 6978. | -14989. | -677. | 1271. | -1689. | -1604. | -1604. | -1605. | -170. | -1604. | -9439. | 0. | 0. | -1604. | 0. | |
| 1.5° | 0. | -2829. | -0. | -529. | 6927. | -821. | -529. | 215. | -1631. | -1631. | -1631. | -1631. | -170. | -1631. | -8457. | 0. | 0. | -1631. | 0. | |
| 1.0° | 0. | -7363. | -0. | -439. | 9967. | -1233. | -439. | 215. | -1666. | -1666. | -1666. | -1666. | -170. | -1666. | -8657. | 0. | 0. | -1666. | 0. | |
| 0.5° | 0. | -10323. | -0. | -331. | 11107. | -6556. | -331. | 0. | -331. | -0. | -676. | -0. | -676. | -0. | -676. | -5545. | -3. | 0. | -676. | 0. |
| 0. | 0. | -11351. | -0. | -304. | -11503. | -0. | -304. | 0. | -304. | -0. | -676. | -0. | -676. | -0. | -676. | -5545. | -3. | 0. | -676. | 0. |
| ANGLE (DEG) | RRAP | TTAP | RRBM | TTBM | TRANVERSE STRESS (LB/IN/IN) FRCN | RTCM | RRCP | TTCP | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | | |
| 4.5° | 0. | -22739. | -0. | -1297. | 1626. | 0. | -11207. | 24. | -1183. | -204. | -1186. | -12952. | -170. | -1183. | -5957. | 0. | 0. | -1183. | 0. | |
| 4.0° | 0. | -21711. | -0. | -11180. | 1230. | -656. | -11205. | 31. | -1178. | -214. | -179. | -12729. | -170. | -1273. | -7074. | 0. | 0. | -1273. | 0. | |
| 3.5° | 0. | -15752. | -0. | -11031. | 1290. | -1232. | -11041. | 53. | -1233. | -226. | -1236. | -12395. | -170. | -1236. | -7253. | 0. | 0. | -1236. | 0. | |
| 3.0° | 0. | -14216. | -0. | -11661. | 1656. | -1261. | -11671. | 36. | -1154. | -214. | -1155. | -11168. | -170. | -1155. | -7589. | 0. | 0. | -1155. | 0. | |
| 2.5° | 0. | -13654. | -0. | -1634. | 1689. | -1284. | -1634. | 36. | -1689. | -216. | -1686. | -1551. | -170. | -1686. | -8731. | 0. | 0. | -1686. | 0. | |
| 2.0° | 0. | -2734. | -0. | -677. | 6978. | -14989. | -677. | 1271. | -1689. | -1604. | -1604. | -1605. | -170. | -1604. | -9439. | 0. | 0. | -1604. | 0. | |
| 1.5° | 0. | -2829. | -0. | -529. | 6927. | -821. | -529. | 215. | -1631. | -1631. | -1631. | -1631. | -170. | -1631. | -8457. | 0. | 0. | -1631. | 0. | |
| 1.0° | 0. | -7363. | -0. | -439. | 9967. | -1233. | -439. | 215. | -1666. | -1666. | -1666. | -1666. | -170. | -1666. | -8657. | 0. | 0. | -1666. | 0. | |
| 0.5° | 0. | -10323. | -0. | -331. | 11107. | -6556. | -331. | 0. | -331. | -0. | -676. | -0. | -676. | -0. | -676. | -5545. | -3. | 0. | -676. | 0. |
| 0. | 0. | -11351. | -0. | -304. | -11503. | -0. | -304. | 0. | -304. | -0. | -676. | -0. | -676. | -0. | -676. | -5545. | -3. | 0. | -676. | 0. |
| ANGLE (DEG) | RRAP | TTAP | RRBM | TTBM | TRANVERSE STRESS (LB/IN/IN) FRCN | RTCM | RRCP | TTCP | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | | |
| 4.5° | 0. | -22739. | -0. | -1297. | 1626. | 0. | -11207. | 24. | -1183. | -204. | -1186. | -12952. | -170. | -1183. | -5957. | 0. | 0. | -1183. | 0. | |
| 4.0° | 0. | -21711. | -0. | -11180. | 1230. | -656. | -11205. | 31. | -1178. | -214. | -179. | -12729. | -170. | -1273. | -7074. | 0. | 0. | -1273. | 0. | |
| 3.5° | 0. | -15752. | -0. | -11031. | 1290. | -1232. | -11041. | 53. | -1233. | -226. | -1236. | -12395. | -170. | -1236. | -7253. | 0. | 0. | -1236. | 0. | |
| 3.0° | 0. | -14216. | -0. | -11661. | 1656. | -1261. | -11671. | 36. | -1154. | -214. | -1155. | -11168. | -170. | -1155. | -7589. | 0. | 0. | -1155. | 0. | |
| 2.5° | 0. | -13654. | -0. | -1634. | 1689. | -1284. | -1634. | 36. | -1689. | -216. | -1686. | -1551. | -170. | -1686. | -8731. | 0. | 0. | -1686. | 0. | |
| 2.0° | 0. | -2734. | -0. | -677. | 6978. | -14989. | -677. | 1271. | -1689. | -1604. | -1604. | -1605. | -170. | -1604. | -9439. | 0. | 0. | -1604. | 0. | |
| 1.5° | 0. | -2829. | -0. | -529. | 6927. | -821. | -529. | 215. | -1631. | -1631. | -1631. | -1631. | -170. | -1631. | -8457. | 0. | 0. | -1631. | 0. | |
| 1.0° | 0. | -7363. | -0. | -439. | 9967. | -1233. | -439. | 215. | -1666. | -1666. | -1666. | -1666. | -170. | -1666. | -8657. | 0. | 0. | -1666. | 0. | |
| 0.5° | 0. | -10323. | -0. | -331. | 11107. | -6556. | -331. | 0. | -331. | -0. | -676. | -0. | -676. | -0. | -676. | -5545. | -3. | 0. | -676. | 0. |
| 0. | 0. | -11351. | -0. | -304. | -11503. | -0. | -304. | 0. | -304. | -0. | -676. | -0. | -676. | -0. | -676. | -5545. | -3. | 0. | -676. | 0. |
| ANGLE (DEG) | RRAP | TTAP | RRBM | TTBM | TRANVERSE STRESS (LB/IN/IN) FRCN | RTCM | RRCP | TTCP | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | RTDM | TTDM | | |
| 4.5° | 0. | -22739. | -0. | -1297. | 1626. | 0. | -11207. | 24. | -1183. | -204. | -1186. | -12952. | -170. | -1183. | -5957. | 0. | 0. | -1183. | 0. | |
| 4.0° | 0. | -21711. | -0. | -11180. | 1230. | -656. | -11205. | 31. | -1178. | -214. | -179. | -12729. | -170. | -1273. | -7074. | 0. | 0. | -1273. | 0. | |
| 3.5° | 0. | -15752. | -0. | -11031. | 1290. | -1232. | -11041. | 53. | -1233. | -226. | -1236. | -12395. | -170. | -1236. | -7253. | 0. | 0. | -1236. | 0. | |
| 3.0° | 0. | -14216. | -0. | -11661. | 1656. | -1261. | -11671. | 36. | -1154. | -214. | -1155. | -11168. | -170. | -1155. | -7589. | 0. | 0. | -1155. | 0. | |
| 2.5° | 0. | -13654. | -0. | -1634. | 1689. | -1284. | -1634. | 36. | -1689. | -216. | -1686. | -1551. | -170. | -1686. | -8731. | 0. | 0. | -1686. | 0. | |
| 2.0° | 0. | -2734. | -0. | -677. | 6978. | -14989. | -677. | 1271. | -1689. | -1604. | -1604. | -1605. | -170. | -1604. | -9439. | 0. | 0. | -1604. | 0. | |
| 1.5° | 0. | -2829. | -0. | -529. | 6927. | -821. | -529. | 215. | -1631. | -1631. | -1631. | -1631. | -170. | -1631. | -8457. | 0. | 0. | -1631. | 0. | |
| 1.0° | 0. | -7363. | -0. | -439. | 9967. | -1233. | -439. | 215. | -1666. | -1666. | -1666. | -1666. | -170. | -1666. | -8657. | 0. | 0. | -1666. | 0. | |
| 0.5° | 0. | -10323. | -0. | -331. | 11107. | -6556. | -331. | 0. | -331. | -0. | -676. | -0. | -676. | -0. | -676. | -5545. | -3. | 0. | -676. | 0. |
| 0. | 0. | -11351. | -0. | -304. | -11503. | -0. | -304. | 0. | -304. | -0. | -676. | -0. | -676. | -0. | -676. | -5545. | -3. | 0. | -676. | 0. |

FN-306
1620

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRIBOL F 15

| | | | | | | | | |
|---|---|-------|-----------------------------------|---|--------|-----------------------------------|---|-------|
| CENTRAL GRADIENT(KG/IN) | = | 0.000 | INNER BORE RADIUS(IN) | = | 3.000 | INNER CONDUCTOR RADIUS(IN) | = | 3.500 |
| CENTER CONDUCTOR RADIUS(LBS/IN) | = | 0.000 | CUTTER RADIUS OF BANDING(CLSS/IN) | = | 6.500 | RADIUS OF IRON SHIELD(CLSS/IN) | = | 6.000 |
| CENTER CONDUCTOR YOUNG'S MODULUS(CLSS/IN) | = | 29000 | COND. YOUNG'S MODULUS(CLSS/IN) | = | 100000 | RADIUS OF OUTERS HOUDLUS(CLSS/IN) | = | 11000 |
| COEFF. POISSON'S RATIO(CLSS/IN) | = | 0.333 | COND. POISSON'S RATIO(CLSS/IN) | = | 0.333 | RADIUS OF OUTERS SHIELD(CLSS/IN) | = | 11332 |
| COEFF. POISSON'S STRAIN(CLSS/IN) | = | 0.333 | COND. THICKNESS STRAIN(CLSS/IN) | = | 0.093 | RADIUS OF OUTERS STATION(CLSS/IN) | = | 11332 |
| DISLOCATON TYPE | = | 200 | MAGNETIC ENERGY(CLIN-LBS)/IN | = | 3.00 | MAGNETIC ENERGY(CL/M) | = | 0.00 |
| DISLOCATON | = | 0.015 | | | | | | |

Case 7. Cool Down, With Pre Load, No Excitation

ELASTIC STRESS AND STRAIN IN SEAM LINE QUADRUPOLE ELQUAD

32
TRANSVERSE STRAIN (MUNINCH)
TRANSDISPACER
PTRM DD70
PAM

Case 8. Cool Down, With Pre Load: With Excitation